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(3) Corresponding to a factor  $a^2$  in the number, we have in the sum of the divisors  $1 + a + a^2$ , etc. By forming a table of  $1 + a + a^2$ ,  $1 + a + a^2 + a^3$ , ... we can find examples where the number contains square factors, cube factors, ...

EXAMPLES:

| $x$             | $\Sigma x$ | $x$        | $\Sigma x$ |
|-----------------|------------|------------|------------|
| $7^2.11^2.5.13$ | $798^2$    | $3^3.7.19$ | $80^2$     |
| $41^3.83$       | $2436^2$   | $2^2.5.41$ | $42^2$     |
| $41^3.5.13$     | $2436^2$   | .          | .          |
| $3^3.89$        | $60^2$     | .          | .          |
|                 |            | .          | .          |

## QUESTIONS AND DISCUSSIONS.

SEND ALL COMMUNICATIONS TO U. G. MITCHELL, University of Kansas, Lawrence, Kans.

### REPLIES.

31. What are the actual courses now offered in colleges and universities in this country for the preparation of teachers (1) for secondary schools, (2) for colleges? The discussion may well lead to the consideration also of what courses *should be offered* for the preparation of teachers of mathematics (1) for secondary schools, and (2) for colleges.

REPLY BY U. G. MITCHELL, University of Kansas.

In the American Report, Committee No. V, of the International Commission on the Teaching of Mathematics, published in 1911, we find (pp. 5-6) these statements:<sup>1</sup>

"Twenty years ago no professional training of university grade existed in this country to prepare teachers of mathematics for secondary schools. At that time the young teacher's sole preparation for his work was the taking of as many academic courses as possible, plus, in some instances, a course on the history of education or some lectures on pedagogy. . . . About 15 years ago we find conditions throughout the country beginning to change in this respect. At least five different educational institutions had by this time (1895) established courses on the teaching of algebra and geometry, which, together with a course on general pedagogy, formed a certain professional training for high school teaching in mathematics. Up to 1900 only four other colleges are known to this committee to have added courses in the pedagogy of secondary mathematics to their programs.

"The past 10 years have shown far greater interest in pedagogical matters and a much more rapid growth in courses of this kind. At present (1910) about 25 other colleges in addition to those above mentioned, have developed such courses."

The writer has just completed an examination of the most recent catalogs from a selected list of 100 colleges and universities to see how many of them are now offering courses especially designed for students preparing to teach mathematics. The result shows that the change referred to above is continuing. Fifty-three of the 100 institutions are offering one or more courses of the following four different classes:

(1) *Courses especially designed for teachers of secondary mathematics.* Such courses were listed by 40 of the 100 institutions and were variously designated as "Teachers' Course," "Mathematical Methods," "History and Teaching of Mathematics," "Teachers' Course in Algebra and Geometry," etc. In five cases "reviews of secondary mathematics with special attention to methods"

<sup>1</sup> *Bulletin, 1911, No. 12, U. S. Bureau of Education* (Washington, D. C.).

constituted a part or all of the course. In a considerable number of cases it was stated that courses in analytic geometry and calculus were prerequisites. The credit given for such courses varied from 1 to 6 hours.

(2) *Courses in practice teaching.* In 6 of the 100 institutions students were offered opportunity to do actual teaching under the direction of supervisors in training schools.

(3) *Courses in the history of mathematics.* Such courses were offered in 24 of the 100 institutions and, with a single exception, were given either two or three hours credit. Two institutions offered more than one such course. Nine of the institutions which gave no separate course in history of mathematics stated that a part of the time of the teachers' course was devoted to history. Hence, some formal teaching in the history of mathematics was offered in at least 33 of the 100 institutions.

(4) *Courses in the foundations of mathematics.* Such courses were offered in 8 of the 100 institutions. They were variously designated as "Fundamental Concepts of Mathematics," "Fundamental Theorems of Algebra and Geometry," "Foundations of Geometry," etc., and were generally three-hour courses.

The writer failed to find any courses which were said to be designed especially to prepare teachers of collegiate mathematics. It is quite possible, however, that some of the history and foundation courses have been established primarily for students who expect to become professors of mathematics. In this respect, the condition is practically the same as described by the American Report, Committee X, of the International Commission on the Teaching of Mathematics in the following language<sup>1</sup> (*italics mine*):

"There is a universal feeling that courses in the pedagogy of mathematics are of very small advantage to the future college teacher. Those professors who are willing to see such courses introduced specify either that they should be in addition to all the present courses in mathematics and not a substitute for any purely mathematical subject, or that they should be introduced only to satisfy the imperative demands of the schools.

"A wiser view seems to be that which argues that the college teacher can well afford to spend some time in learning the best methods of teaching, whatever his subject; and that a course in the pedagogy of mathematics will be of very great value to the teacher of mathematics when the graduate school or university shall have developed a course suited to his needs. *There appears to be sufficient material at hand, and it is strange that no American institution has solved the problem of a course in mathematical pedagogy in such a manner as to appeal to the professor of mathematics.*"

As to what courses should be offered for the preparation of teachers of secondary mathematics, a considerable number of American institutions are now offering and, in large part, requiring, the following standard suggested as a minimum by Professor J. W. A. Young at the Cambridge meeting<sup>2</sup> (1912) of the International Commission:

- (a) Trigonometry, college algebra, analytic geometry.
- (b) Surveying, or descriptive geometry, or elementary astronomy.
- (c) The differential and integral calculus with applications to geometry, mechanics and physics.
- (d) Modern geometry.

<sup>1</sup> *Bulletin*, 1911, No. 7, U. S. Bureau of Education (Washington, D. C.), pp. 23-4.

<sup>2</sup> See *L'Enseignement Mathématique* for Nov. 15, 1912, p. 483.

- (e) The elements of analytic mechanics.
- (f) The elements of theoretic and laboratory physics.
- (g) Algebra from a modern standpoint.
- (h) One or more courses introductory to important fields of modern mathematics.
- (i) One or more courses on the history of mathematics.
- (j) One or more courses on the teaching of mathematics.

At the university with which the writer is connected, a student majoring in mathematics is required to take before graduation at least 30 hours of work in pure mathematics courses. Besides the usual courses in college algebra, trigonometry, analytical geometry and calculus (15 hours), a two-hour course in higher algebra, a three-hour course in modern geometry, a two-hour course in history of mathematics and a three-hour course in analytic mechanics or advanced calculus, are specifically required. The remaining 5 or more hours are elective. In order to be certificated for teaching mathematics in the secondary schools in Kansas the student must also take a three-hour teachers' course in mathematics, a semester's work in practice teaching in the training school, a three-hour course in general psychology, a three-hour course in educational psychology, a three-hour course in the history of education and 6 hours of elective work in the school of education. While these requirements are probably less than some others, they doubtless do not differ greatly from those of a number of middle-western universities.

The writer believes that courses in education possess a distinct value for the prospective teacher of mathematics. They give him some conception of the human side of his work. More than one young professor, splendidly equipped mathematically, has failed as a teacher because he spent his energy in trying to teach mathematics *to* students and never acquired the point of view of teaching students *by means of* mathematics.

REPLY BY R. C. ARCHIBALD, Brown University.

Within the past decade many of those charged with directing the education of youth in the United States have had their outlook immensely extended, and their ideas radically changed by careful study and consideration of methods employed in other countries. They have discovered that the standards to which teachers in practically all the secondary schools of France, Germany, Italy and many other countries must attain are far higher than those demanded in even the best secondary schools of the United States. As far as mathematics is concerned this has been set forth with great explicitness, during the past few years through the publications inspired by the International Commission on the Teaching of Mathematics.

As a result, thoughtful inquirers have reached the conclusion that the efficiency and general well-being of this country demand that some radical changes be made in the method of conducting secondary education; moreover, that as a beginning of necessary reform, newly appointed teachers should be college graduates who are informed as to the problems and methods in secondary education and who, while they have learned to appreciate varied forms of scholarship and culture, have yet been specially equipped to teach one or two nearly related subjects.

It is with deep appreciation of the important work to be done in promoting the cause of secondary education, and therefore of higher education, that the department of mathematics in Brown University lays emphasis, by virtue of special courses and personal effort, on the methods employed in the preparation of teachers of mathematics. The annual increase in the number of students taking this work in the department seems to testify to the fact that a real need is being met.

It is considered important that prospective teachers of mathematics should have a thorough scientific reconsideration in college of the principles of secondary school mathematics, that the true inwardness of all operations, and a clear understanding of the foundations of the various subjects, should be acquired. This knowledge, together with adequate facility in presenting it, presupposes that the student has been carried far beyond the subject matter actually to be taught at a later day in the secondary school. But it is only with such preparation that a teacher of mathematics can claim to be competent to lay the foundations of a subject which is doubtless of more far-reaching importance than any other.

The eleven semester courses (each three hours per week) required for recommended teachers are: algebra, geometry, plane trigonometry, analytical geometry, and differential and integral calculus.

The two-semester *teachers' course in algebra* constitutes an introduction to some of the concepts of modern analysis. Among the topics treated are, the number system, with special reference to irrational numbers, limits, infinite series, fundamental operations, determinants and proofs of the fundamental theorem of algebra.

In the two-semester *teachers' course in elementary geometry* the student is taught: methods for attacking Euclidean problems; discussions of famous problems; the existence of transcendental numbers and the proof of the transcendence of  $\pi$ ; means of rigorous discussion of the more delicate and difficult parts of the subject such as the systems of axioms; something about (a) the history and literature of elementary geometry and (b) the most important French, German and Italian texts.

As to the course in *solid and spherical geometry*, the ability to reason accurately from fundamental axioms, and to solve originals, is demanded.

The work in *analytical geometry* occupies one and one-half semesters and that in *calculus* two and one-half semesters.

In addition to those mentioned above it is possible to take six or eight other year-courses, partaking somewhat of a graduate character, and some such courses are usually elected by our embryonic secondary school teachers. For example, during the year 1915-16 Goursat's *Mathematical Analysis*, volume 1, was a text in one course; in another, Pierpont's *Functions of a Complex Variable*, and in a third, Papelier's *Coordonnées tangentielles*. In no one of these courses were there less than ten students, the majority of whom were preparing to be secondary school teachers. For the year 1916-17, the first volume of Weber's

*Algebra* is a text in one course and in another, in differential geometry, the texts are the last part of volume 1 of Goursat's *Mathematical Analysis*, Gauss's memoirs on *Curved Surfaces* and selected chapters from Humbert's *Cours d'Analyse*.

Another advantage in connection with our preparation of teachers is the meetings of the Mathematical Club. The club is a recent organization which aims to create an atmosphere in which to establish friendly intercourse among students and teachers, and to supply an opportunity for the presentation, by advanced students and professors, of papers of general interest.

Practically all students recommended as teachers of mathematics to secondary schools have had two or more courses in education besides work in science. Many students return for graduate work leading to the degree of Master of Arts. Their work is mainly in the department of education and in the schools of Providence and the surrounding cities. Under suitable restrictions tuition fees are met through scholarships awarded by the Board of Education of the state of Rhode Island. Such students (men and women) are not only well equipped scholastically, but have also acquired valuable insight and experience with reference to methods of imparting knowledge.

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## NOTES AND NEWS.

SEND ALL COMMUNICATIONS TO D. A. ROTHROCK, Indiana University.

The distinguished mathematical physicist, Professor DON JOSÉ ECHEGARAY, of the University of Madrid, died on Sept. 15, aged eighty-three years.

Mr. F. S. NOWLAN, of Columbia University, has been appointed instructor in mathematics at the Carnegie School of Technology, Pittsburgh.

At Pennsylvania State College, Dr. J. E. ROWE has been promoted from assistant professor to associate professor of mathematics.

Mr. E. J. OGLESBY, instructor in mathematics in the University of Virginia, has been appointed professor of mathematics at the College of William and Mary.

Mr. ALBERT H. HOLMES, a contributor to the MONTHLY in the department of Problems and Solutions, and a charter member of the Association, died at Brunswick, Maine, Sept. 10, 1916, at the age of sixty-five years.

Dr. J. A. BULLARD, Mr. C. E. NORWOOD, and Mr. J. J. TANZOLA have been appointed instructors in mathematics at the U. S. Naval Academy, Annapolis, Maryland.

At Dartmouth College, Dr. R. D. BEETLE and Dr. F. M. MORGAN have been made assistant professors of mathematics, and Dr. C. H. FORSYTH, of the University of Michigan, has been appointed instructor in mathematics.